Rate Measures

The truncated Z methodology for rate measures has the same general structure for calculating the Z in each cell as proportion measures. For a rate measure, there are a fixed number of circuits or units for the CLEC, n_{2j} and a fixed number of units for BST, n_{1j}. Suppose that the performance measure is a "trouble rate." The modeling assumption is that the occurrence of a trouble is independent between units and the number of troubles in n circuits follows a Poisson distribution with mean λ_n where λ is the probability of a trouble in 1 circuit and n is the number of circuits.

In an adjustment cell, if the number of CLEC troubles is greater than 15 and the number of BST troubles is greater than 15, then the Z test is calculated using the normal approximation to the Poisson. In this case, the moments of the truncated Z come directly from properties of the standard normal distribution. Otherwise, if there are very few troubles, the number of CLEC troubles can be modeled using a binomial distribution with n equal to the total number of troubles (CLEC plus BST troubles.) In this case, the moments for the truncated Z are calculated explicitly using the binomial distribution.

Mean Measures

For mean measures, an adjusted "t" statistic is calculated for each like-to-like cell which has at least 7 BST and 7 CLEC transactions. A permutation test is used when one or both of the BST and CLEC sample sizes is less than 6. Both the adjusted "t" statistic and the permutation calculation are described in Appendix D, Statistical Formulas and Technical Description.

Ratio Measures

Rules will be given for computing a cell test statistic for a ratio measure, however, the current plan for measures in this category, namely billing accuracy, does not call for the use of a Z parity statistic.

Appendix D: Statistical Formulas and Technical Description

We start by assuming that any necessary trimming[1] of the data is complete, and that the data are disaggregated so that comparisons are made within appropriate classes or adjustment cells that define "like" observations.

1. Notation and Exact Testing Distributions

Below, we have detailed the basic notation for the construction of the truncated z statistic. In what follows the word "cell" should be taken to mean a like-to-like comparison cell that has both one (or more) ILEC observation and one (or more) CLEC observation.

$L =$	the total number of occupied cells
$j =$	$1,\ldots,L$; an index for the cells
$n_{1j} =$	the number of ILEC transactions in cell j
$n_{2j} =$	the number of CLEC transactions in cell j
$n_j =$	the total number transactions in cell j; $n_{1j} + n_{2j}$
$X_{1jk} =$	individual ILEC transactions in cell j; $k = 1,\ldots, n_{1j}$
$X_{2jk} =$	individual CLEC transactions in cell j; $k = 1,\ldots, n_{2j}$
$Y_{jk} =$	individual transaction (both ILEC and CLEC) in cell j
	$= \begin{cases} X_{1jk} & k = 1,\ldots,n_{1j} \\ X_{2jk} & k = n_{1j}+1,\ldots,n_j \end{cases}$
$\Phi^{-1}(\cdot) =$	the inverse of the cumulative standard normal distribution function

1. When it is determined that a measure should be trimmed, a trimming rule that is easy to implement in a production setting is:

Trim the ILEC observations to the largest CLEC value from all CLEC observations in the month under consideration.

That is, no CLEC values are removed; all ILEC observations greater than the largest CLEC observation are trimmed.

For Mean Performance Measures the following additional notation is needed.

$\overline{X}_{1j}$ = The ILEC sample mean of cell j

$\overline{X}_{2j}$ = The CLEC sample mean of cell j

s_{1j}^2 = The ILEC sample variance in cell j

s_{2j}^2 = The CLEC sample variance in cell j

$\{y_{jk}\}$ = a random sample of size n_{2j} from the set of $Y_{j1},\ldots,Y_{jn_j}$; $k = 1,\ldots,n_{2j}$

M_j = The total number of distinct pairs of samples of size n_{1j} and n_{2j};

$$= \binom{n_j}{n_{1j}}$$

The exact parity test is the permutation test based on the "modified Z" statistic. For large samples, we can avoid permutation calculations since this statistic will be normal (or Student's t) to a good approximation. For small samples, where we cannot avoid permutation calculations, we have found that the difference between "modified Z" and the textbook "pooled Z" is negligible. We therefore propose to use the permutation test based on pooled Z for small samples. This decision speeds up the permutation computations considerably, because for each permutation we need only compute the sum of the CLEC sample values, and not the pooled statistic itself.

A permutation probability mass function distribution for cell j, based on the "pooled Z" can be written as

$$PM(t) = P(\sum_k y_{jk} = t) = \frac{\textit{the number of samples that sum to } t}{M_j}$$

and the corresponding cumulative permutation distribution is

$$CPM(t) = P(\sum_k y_{jk} \le t) = \frac{\textit{the number of samples with sum } \le t}{M_j}$$

For Proportion Performance Measures the following notation is defined

a_{1j} = The number of ILEC cases possessing an attribute of interest in cell j

a_{2j} = The number of CLEC cases possessing an attribute of interest in cell j

a_j = The number of cases possessing an attribute of interest in cell j; $a_{1j} + a_{2j}$

The exact distribution for a parity test is the hypergeometric distribution. The hypergeometric probability mass function distribution for cell j is

$$HG(h) = P(H = h) = \begin{cases} \dfrac{\dbinom{n_{1j}}{h}\dbinom{n_{2j}}{a_j - h}}{\dbinom{n_j}{a_j}}, & \max(0, a_j - n_{2j}) \le h \le \min(a_j, n_{1j}) \\ 0 & \text{otherwise} \end{cases}$$

and the cumulative hypergeometric distribution is

$$CHG(x) = P(H \le x) = \begin{cases} 0 & x < \max(0, a_j - n_{2j}) \\ \displaystyle\sum_{h=\max(0, a_j - n_{1j})}^{x} HG(h), & \max(0, a_j - n_{2j}) \le x \le \min(a_j, n_{1j}) \\ 1 & x > \min(a_j, n_{1j}) \end{cases}$$

For Rate Measures, the notation needed is defined as

b_{1j} = The number of ILEC base elements in cell j

b_{2j} = The number of CLEC base elements in cell j

b_j = The total number of base elements in cell j; $b_{1j} + b_{2j}$

$\hat{r}_{1j}$ = The ILEC sample rate of cell j; n_{1j}/b_{1j}

$\hat{r}_{2j}$ = The CLEC sample rate of cell j; n_{2j}/b_{2j}

q_j = The relative proportion of ILEC elements for cell j; b_{1j}/b_j

The exact distribution for a parity test is the binomial distribution. The binomial probability mass function distribution for cell j is

$$BN(k) = P(B = k) = \begin{cases} \binom{n_j}{k} q_j^k (1 - q_j)^{n_j - k}, & 0 \le k \le n_j \\ 0 & \text{otherwise} \end{cases}$$

and the cumulative binomial distribution is

$$CBN(x) = P(B \le x) = \begin{cases} 0 & x < 0 \\ \sum_{k=0}^{x} BN(k), & 0 \le x \le n_j \\ 1 & x > n_j \end{cases}$$

For Ratio Performance Measures the following additional notation is needed.

U_{1jk} = additional quantity of interest of an individual ILEC transaction in cell j; $k = 1,\ldots, n_{1j}$

U_{2jk} = additional quantity of interest of an individual CLEC transaction in cell j; $k = 1,\ldots, n_{2j}$

$\hat{R}_{ij}$ = the ILEC (I = 1) or CLEC (i = 2) ratio of the total additional quantity of interest to the base transaction total in cell j, i.e.,
$\sum_k U_{ijk} / \sum_k X_{ijk}$

2. Calculating the Truncated Z

The general methodology for calculating an aggregate level test statistic is outlined below.

Calculate Cell Weights (W_j)

A weight based on the number of transactions is used so that a cell, which has a larger number of transactions, has a larger weight. The actual weight formulae will depend on the type of measure.

Mean or Ratio Measure

$$W_j = \sqrt{\frac{n_{1j} n_{2j}}{n_j}}$$

Proportion Measure

$$W_j = \sqrt{\frac{n_{2j} n_{1j}}{n_j} \cdot \frac{a_j}{n_j} \cdot \left(1 - \frac{a_j}{n_j}\right)}$$

Rate Measure

$$W_j = \sqrt{\frac{b_{1j} b_{2j}}{b_j} \cdot \frac{n_j}{b_j}}$$

Calculate a Z Value (Z_j) for each Cell

A Z statistic with mean 0 and variance 1 is needed for each cell.

- If $W_j = 0$, set $Z_j = 0$.
- Otherwise, the actual Z statistic calculation depends on the type of performance measure.

Mean Measure

$Z_j = \Phi^{-1}(\alpha)$

where α is determined by the following algorithm.

If $\min(n_{1j}, n_{2j}) > 6$, then determine α as

$$\alpha = P(t_{n_{1j}-1} \le T_j)$$

that is, α is the probability that a t random variable with $n_{1j} - 1$ degrees of freedom, is less than

$$T_j = \begin{cases} t_j + \dfrac{g}{6}\left(\dfrac{n_{1j} + 2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j} + n_{2j})}}\right)\left(t_j^2 + \dfrac{n_{2j} - n_{1j}}{n_{1j} + 2n_{2j}}\right) & t_j \ge t_{\min j} \\[2ex] t_j + \dfrac{g}{6}\left(\dfrac{n_{1j} + 2n_{2j}}{\sqrt{n_{1j}\, n_{2j}(n_{1j} + n_{2j})}}\right)\left(t_{\min j}^2 + \dfrac{n_{2j} - n_{1j}}{n_{1j} + 2n_{2j}}\right) & \text{otherwise} \end{cases}$$

where

$$t_j = \frac{\bar{X}_{1j} - \bar{X}_{2j}}{s_{1j}\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}}$$

$$t_{\min j} = \frac{-3\sqrt{n_{1j} n_{2j} n_j}}{g(n_{1j} + 2n_{2j})}$$

and g is the median value of all values of

$$\gamma_{1j} = \frac{n_{1j}}{(n_{1j}-1)(n_{1j}-2)} \sum_k \left(\frac{X_{1jk} - \bar{X}_{1j}}{s_{1j}} \right)^3$$

with $n_{1j} > n_{3q}$ for all values of j. n_{3q} is the 3 quartile of all values of n_{1j}

Note, that t_j is the "modified Z" statistic. The statistic T_j is a "modified Z" corrected for the skewness of the ILEC data.

If $\min(n_{1j}, n_{2j}) \le 6$, and

- $M_j \le 1{,}000$ (the total number of distinct pairs of samples of size n_{1j} and n_{2j} is 1,000 or less).
  - Calculate the sample sum for all possible samples of size n_{2j}.
  - Rank the sample sums from smallest to largest. Ties are dealt by using average ranks.
  - Let R_0 be the rank of the observed sample sum with respect all the sample sums.

$$\alpha = 1 - \frac{R_0 - 0.5}{M_j}$$

- b) $M_j > 1{,}000$
  - Draw a random sample of 1,000 sample sums from the permutation distribution.
  - Add the observed sample sum to the list. There are a total of 1001 sample sums. Rank the sample sums from smallest to largest. Ties are dealt by using average ranks.
  - Let R_0 be the rank of the observed sample sum with respect all the sample sums.

$$\alpha = 1 - \frac{R_0 - 0.5}{1001}$$

Proportion Measure

$$Z_j = \frac{n_j a_{1j} - n_{1j} a_j}{\sqrt{\frac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}$$

Rate Measure

$$Z_j = \frac{n_{1j} - n_j q_j}{\sqrt{n_j q_j (1 - q_j)}}$$

Ratio Measure

$$Z_j = \frac{\hat{R}_{1j} - \hat{R}_{2j}}{\sqrt{V(\hat{R}_{1j})\left(\frac{1}{n_{1j}} + \frac{1}{n_{2j}}\right)}}$$

$$V(\hat{R}_{1j}) = \frac{\sum_k \left(U_{1jk} - \hat{R}_{1j} X_{1jk}\right)^2}{\bar{X}_{1j}^2 (n_{1j} - 1)} = \frac{\sum_k U_{1jk}^2 - 2\hat{R}_{1j} \sum_k \left(U_{1jk} X_{1jk}\right) + \hat{R}_{1j}^2 \sum_k X_{1jk}^2}{\bar{X}_{1j}^2 (n_{1j} - 1)}$$

Obtain a Truncated Z Value for each Cell (Z^*_j)

To limit the amount of cancellation that takes place between cell results during aggregation, cells whose results suggest possible favoritism are left alone. Otherwise the cell statistic is set to zero. This means that positive equivalent Z values are set to 0, and negative values are left alone. Mathematically, this is written as

$$Z_j^* = \min(0, Z_j)$$

Calculate the Theoretical Mean and Variance

Calculate the theoretical mean and variance of the truncated statistic under the null hypothesis of parity, $E(Z_j^* \mid H_0)$ and $Var(Z_j^* \mid H_0)$. To compensate for the truncation in step 3, an aggregated, weighted sum of the Z^*_j will need to be centered and scaled properly so that the final aggregate statistic follows a standard normal distribution.

- If $W_j = 0$, then no evidence of favoritism is contained in the cell. The formulae for calculating $E(Z_j^* \mid H_0)$ and $Var(Z_j^* \mid H_0)$ cannot be used. Set both equal to 0.
- If $\min(n_{1j}, n_{2j}) > 6$ for a mean measure, $\min\left\{a_{1j}\left(1-\frac{a_{1j}}{n_{1j}}\right), a_{2j}\left(1-\frac{a_{2j}}{n_{2j}}\right)\right\} > 9$ for a proportion measure, $\min(n_{1j}, n_{2j}) > 15$ and $n_j q_j (1-q_j) > 9$ for a rate measure, or n_{1j} and n_{2j} are large for a ratio measure then

$$E(Z_j^* \mid H_0) = -\frac{1}{\sqrt{2\pi}}$$

and

$$Var(Z_j^* \mid H_0) = \frac{1}{2} - \frac{1}{2\pi}$$

- Otherwise, determine the total number of values for Z^*_j. Let z_{ji} and θ_{ji}, denote the values of Z^*_j and the probabilities of observing each value, respectively.

$$E(Z_j^* \mid H_0) = \sum_i \theta_{ji} z_{ji}$$

and

$$Var(Z_j^* \mid H_0) = \sum_i \theta_{ji} z_{ji}^2 - \left[E(Z_j^* \mid H_0)\right]^2$$

The actual values of the z's and θ's depends on the type of measure.

Mean Measure

$$N_j = \min(M_j, 1{,}000), \quad i = 1, \ldots, N_j$$

$$z_{ji} = \min\left\{0, \Phi^{-1}\left(1 - \tfrac{R_i - 0.5}{N_j}\right)\right\} \quad \text{where } R_i \text{ is the rank of sample sum i}$$

$$\theta_j = \frac{1}{N_j}$$

Proportion Measure

$$z_{ji} = \min\left\{0, \frac{n_j\, i - n_{1j}\, a_j}{\sqrt{\dfrac{n_{1j}\, n_{2j}\, a_j\,(n_j - a_j)}{n_j - 1}}}\right\}, \quad i = \max(0, a_j - n_{2j}), \ldots, \min(a_j, n_{1j})$$

$$\theta_{ji} = HG(i)$$

Rate Measure

$$z_{ji} = \min\left\{0, \frac{i - n_j\, q_j}{\sqrt{n_j\, q_j (1 - q_j)}}\right\}, \quad i = 0, \ldots, n_j$$

$$\theta_{ji} = BN(i)$$

Ratio Measure

The performance measure that is in this class is billing accuracy. If a parity test were used, the sample sizes for this measure are quite large, so there is no need for a small sample technique. If one does need a small sample technique, then a re-sampling method can be used.

Calculate the Aggregate Test Statistic (Z^T)

$$Z^T = \frac{\sum_j W_j Z_j^* - \sum_j W_j E(Z_j^* \mid H_0)}{\sqrt{\sum_j W_j^2 \operatorname{Var}(Z_j^* \mid H_0)}}$$

The Balancing Critical Value

There are four key elements of the statistical testing process:

- the null hypothesis, H_0, that parity exists between ILEC and CLEC services
- the alternative hypothesis, H_a, that the ILEC is giving better service to its own customers
- the Truncated Z test statistic, Z^T, and
- a critical value, c

The decision rule[2] is

•If	$Z^T < c$	then	accept H_a.
•If	$Z^T \geq c$	then	accept H_0.

There are two types of error possible when using such a decision rule:

- **Type I Error**: Deciding favoritism exists when there is, in fact, no favoritism.
- **Type II Error**: Deciding parity exists when there is, in fact, favoritism.

The probabilities of each type of each are:

- **Type I Error**: $\alpha = P(Z^T < c \mid H_0)$
- **Type II Error**: $\beta = P(Z^T \geq c \mid H_a)$

We want a balancing critical value, c_B, so that $\alpha = \beta$.

It can be shown that.

$$c_B = \frac{\sum_j W_j M(m_j, se_j) - \sum_j W_j \frac{-1}{\sqrt{2\pi}}}{\sqrt{\sum_j W_j^2 V(m_j, se_j)} + \sqrt{\sum_j W_j^2 \left(\frac{1}{2} - \frac{1}{2\pi}\right)}}$$

2. This decision rule assumes that a negative test statistic indicates poor service for the CLEC customer. If the opposite is true, then reverse the decision rule.

where

$$M(\mu,\sigma) = \mu\,\Phi(\tfrac{-\mu}{\sigma}) - \sigma\,\phi(\tfrac{-\mu}{\sigma})$$

$$V(\mu,\sigma) = (\mu^2 + \sigma^2)\Phi(\tfrac{-\mu}{\sigma}) - \mu\,\sigma\,\phi(\tfrac{-\mu}{\sigma}) - M(\mu,\sigma)^2$$

$\Phi(\cdot)$ is the cumulative standard normal distribution function, and $\phi(\cdot)$ is the standard normal density function.

This formula assumes that Z_j is approximately normally distributed within cell j. When the cell sample sizes, n_{1j} and n_{2j}, are small this may not be true. It is possible to determine the cell mean and variance under the null hypothesis when the cell sample sizes are small. It is much more difficult to determine these values under the alternative hypothesis. Since the cell weight, W_j will also be small (see calculate weights section above) for a cell with small volume, the cell mean and variance will not contribute much to the weighted sum. Therefore, the above formula provides a reasonable approximation to the balancing critical value.

The values of m_j and se_j will depend on the type of performance measure.

Mean Measure

For mean measures, one is concerned with two parameters in each cell, namely, the mean and variance. A possible lack of parity may be due to a difference in cell means, and/or a difference in cell variances. One possible set of hypotheses that capture this notion, and take into account the assumption that transaction are identically distributed within cells is:

H_0: $\mu_{1j} = \mu_{2j}$, $\sigma_{1j}^2 = \sigma_{2j}^2$
H_a: $\mu_{2j} = \mu_{1j} + \delta_j \cdot \sigma_{1j}$, $\sigma_{2j}^2 = \lambda_j \cdot \sigma_{1j}^2$ $\quad$ $\delta_j > 0$, $\lambda_j \geq 1$ and $j = 1,\ldots,L$.

Under this form of alternative hypothesis, the cell test statistic Z_j has mean and standard error given by

$$m_j = \frac{-\delta_j}{\sqrt{\frac{1}{n_{1j}} + \frac{1}{n_{2j}}}}$$

and

$$se_j = \sqrt{\frac{\lambda_j n_{1j} + n_{2j}}{n_{1j} + n_{2j}}}$$

Proportion Measure

For a proportion measure there is only one parameter of interest in each cell, the proportion of transaction possessing an attribute of interest. A possible lack of parity may be due to a difference in cell proportions. A set of hypotheses that take into account the assumption that transaction are identically distributed within cells while allowing for an analytically tractable solution is:

$$H_0: \frac{p_{2j}(1-p_{1j})}{(1-p_{2j})p_{1j}} = 1$$

$$H_a: \frac{p_{2j}(1-p_{1j})}{(1-p_{2j})p_{1j}} = \psi_j \qquad \psi_j > 1 \text{ and } j = 1,\ldots,L.$$

These hypotheses are based on the "odds ratio." If the transaction attribute of interest is a missed trouble repair, then an interpretation of the alternative hypothesis is that a CLEC trouble repair appointment is ψ_j times more likely to be missed than an ILEC trouble.

Under this form of alternative hypothesis, the within cell asymptotic mean and variance of a_{1j} are given by[3]

$$E(a_{1j}) = n_j \pi_j^{(1)}$$

$$\operatorname{var}(a_{1j}) = \frac{n_j}{\frac{1}{\pi_j^{(1)}} + \frac{1}{\pi_j^{(2)}} + \frac{1}{\pi_j^{(3)}} + \frac{1}{\pi_j^{(4)}}}$$

where

$$\pi_j^{(1)} = f_j^{(1)}\left(n_j^2 + f_j^{(2)} + f_j^{(3)} - f_j^{(4)}\right)$$

$$\pi_j^{(2)} = f_j^{(1)}\left(-n_j^2 - f_j^{(2)} + f_j^{(3)} + f_j^{(4)}\right)$$

$$\pi_j^{(3)} = f_j^{(1)}\left(-n_j^2 + f_j^{(2)} - f_j^{(3)} + f_j^{(4)}\right)$$

$$\pi_j^{(4)} = f_j^{(1)}\left(n_j^2\left(\frac{2}{\psi_j} - 1\right) - f_j^{(2)} - f_j^{(3)} - f_j^{(4)}\right)$$

$$f_j^{(1)} = \frac{1}{2n_j^2\left(\frac{1}{\psi_j} - 1\right)}$$

$$f_j^{(2)} = n_j n_{1j}\left(\frac{1}{\psi_j} - 1\right)$$

$$f_j^{(3)} = n_j a_j\left(\frac{1}{\psi_j} - 1\right)$$

$$f_j^{(4)} = \sqrt{n_j^2\left[4n_{1j}(n_j - a_j)\left(\frac{1}{\psi_j} - 1\right) + \left(n_j + (a_j - n_{1j})\left(\frac{1}{\psi_j} - 1\right)\right)^2\right]}$$

3. Stevens, W. L. (1951) Mean and Variance of an entry in a Contingency Table. *Biometrica*, 38, 468-470.

Recall that the cell test statistic is given by

$$Z_j = \frac{n_j a_{1j} - n_{1j} a_j}{\sqrt{\dfrac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}$$

Using the equations above, we see that Z_j has mean and standard error given by

$$m_j = \frac{n_j^2 \pi_j^{(1)} - n_{1j} a_j}{\sqrt{\dfrac{n_{1j} n_{2j} a_j (n_j - a_j)}{n_j - 1}}}$$

and

$$se_j = \sqrt{\frac{n_j^3 (n_j - 1)}{n_{1j} n_{2j} a_j (n_j - a_j)\left(\frac{1}{\pi_j^{(1)}} + \frac{1}{\pi_j^{(2)}} + \frac{1}{\pi_j^{(3)}} + \frac{1}{\pi_j^{(4)}}\right)}}$$

Rate Measure

A rate measure also has only one parameter of interest in each cell, the rate at which a phenomenon is observed relative to a base unit, e.g. the number of troubles per available line. A possible lack of parity may be due to a difference in cell rates. A set of hypotheses that take into account the assumption that transaction are identically distributed within cells is:

H_0: $r_{1j} = r_{2j}$

H_a: $r_{2j} = \varepsilon_j r_{1j}$ $\quad$ $\varepsilon_j > 1$ and $j = 1,\ldots,L$.

Given the total number of ILEC and CLEC transactions in a cell, n_j, and the number of base elements, b_{1j} and b_{2j}, the number of ILEC transaction, n_{1j}, has a binomial distribution from n_j trials and a probability of

$$q_j^* = \frac{r_{1j} b_{1j}}{r_{1j} b_{1j} + r_{2j} b_{2j}}$$

Therefore, the mean and variance of n_{1j}, are given by

$$E(n_{1j}) = n_j q_j^*$$
$$\operatorname{var}(n_{1j}) = n_j q_j^*(1 - q_j^*)$$

Under the null hypothesis

$$q_j^* = q_j = \frac{b_{1j}}{b_j}$$

but under the alternative hypothesis

$$q_j^* = q_j^a = \frac{b_{1j}}{b_{1j} + \varepsilon_j b_{2j}}$$

Recall that the cell test statistic is given by

$$Z_j = \frac{n_{1j} - n_j\, q_j}{\sqrt{n_j\, q_j(1 - q_j)}}$$

Using the relationships above, we see that Z_j has mean and standard error given by

$$m_j = \frac{n_j\left(q_j^a - q_j\right)}{\sqrt{n_j q_j(1 - q_j)}} = (1 - \varepsilon_j)\frac{\sqrt{n_j b_{1j} b_{2j}}}{b_{1j} + \varepsilon_j b_{2j}}$$

and

$$se_j = \sqrt{\frac{q_j^a(1 - q_j^a)}{q_j(1 - q_j)}} = \sqrt{\varepsilon_j}\,\frac{b_j}{b_{1j} + \varepsilon_j b_{2j}}$$

Ratio Measure

As with mean measures, one is concerned with two parameters in each cell, the mean and variance, when testing for parity of ratio measures. As long as sample sizes are large, as in the case of billing accuracy, the same method for finding m_j and se_j that is used for mean measures can be used for ratio measures.

Determining the Parameters of the Alternative Hypothesis

In this section we have indexed the alternative hypothesis of mean measures by two sets of parameters, λ_j and δ_j. Proportion and rate measures have been indexed by one set of parameters each, ψ_j and ε_j respectively. A major difficulty with this approach is that more than one alternative will be of interest; for example we may consider one alternative in which all the δ_j are set to a common non-zero value, and another set of alternatives in each of which just one δ_j is non-zero, while all the rest are zero. There are very many other possibilities. Each possibility leads to a single value for the balancing critical value; and each possible critical value corresponds to many sets of alternative hypotheses, for each of which it constitutes the correct balancing value.

The formulas we have presented can be used to evaluate the impact of different choices of the overall critical value. For each putative choice, we can evaluate the set of alternatives for which this is the correct balancing value. While statistical science can be used to evaluate the impact of different choices of these parameters, there is not much that an appeal to statistical principles can offer in directing specific choices. Specific choices are best left to telephony experts. Still, it is possible to comment on some aspects of these choices:

Parameter Choices for λ_j – The set of parameters λ_j index alternatives to the null hypothesis that arise because there might be greater unpredictability or variability in the delivery of service to a CLEC customer over that which would be achieved for an otherwise comparable ILEC customer. While concerns about differences in the variability of service are important, it turns out that the truncated Z testing which is being recommended here is relatively insensitive to all but very large values of the λ_j. Put another way, reasonable differences in the values chosen here could make very little difference in the balancing points chosen.

Parameter Choices for δ_j – The set of parameters δ_j are much more important in the choice of the balancing point than was true for the λ_j. The reason for this is that they directly index differences in average service. The truncated Z test is very sensitive to any such differences; hence, even small disagreements among experts in the choice of the δ_j could be very important. Sample size matters here too. For example, setting all the δ_j to a single value – $\delta_j = \delta \angle$ might be fine for tests across individual CLECs where currently in Georgia the CLEC customer bases are not too different. Using the same value of δ for the overall state testing does not seem sensible. At the state level we are aggregating over CLECs, so using the same δ as for an individual CLEC would be saying that a "meaningful" degree of disparity is one where the violation is the same (δ) for each CLEC. But the detection of disparity for any component CLEC is important, so the relevant "overall" δ should be smaller.

Parameter Choices for ψ_j *or* ε_j – The set of parameters ψ_j or ε_j are also important in the choice of the balancing point for tests of their respective measures. The reason for this is that they directly index increases in the proportion or rate of service performance. The truncated Z test is sensitive to such increases; but not as sensitive as the case of δ for mean measures. Sample size matters here too. As with mean measures, using the same value of ψ or ε for the overall state testing does not seem sensible.

The three parameters are related however. If a decision is made on the value of δ, it is possible to determine equivalent values of ψ and ε. The following equations, in conjunction with the definitions of ψ and ε, show the relationship with delta.

$$\delta = 2 \cdot \arcsin(\sqrt{\hat{p}_2}) - 2 \cdot \arcsin(\sqrt{\hat{p}_1})$$
$$\delta = 2\sqrt{\hat{r}_2} - 2\sqrt{\hat{r}_1}$$

The bottom line here is that beyond a few general considerations, like those given above, a principled approach to the choice of the alternative hypotheses to guard against must come from elsewhere.

Decision Process

Once Z^T has been calculated, it is compared to the balancing critical value to determine if the ILEC is favoring its own customers over a CLEC's customers.

This critical value changes as the ILEC and CLEC transaction volume change. One way to make this transparent to the decision-maker, is to report the difference between the test statistic and the critical value, *diff* = $Z^T - c_B$. If favoritism is concluded when $Z^T < c_B$, then the *diff* < 0 indicates favoritism.

This makes it very easy to determine favoritism: a positive *diff* suggests no favoritism, and a negative *diff* suggests favoritism.

Appendix E: BST SEEM Remedy Calculation Procedures

BST SEEM Remedy Procedure

1. Tier-1 Calculation For Retail Analogues

1. Calculate the overall test statistic for each CLEC; $z^{T}_{CLEC\text{-}1}$ (Per Statistical Methodology - by Dr. Mulrow)
2. Calculate the balancing critical value (${}^{c}B_{CLEC\text{-}1}$) that is associated with the alternative hypothesis (for fixed parameters δ, Ψ, or ε)
3. If the overall test statistic is equal to or above the balancing critical value, stop here. That is, if ${}^{c}B_{CLEC\text{-}1} < z^{T}_{CLEC\text{-}1}$, stop here. Otherwise, go to step 4.
4. Calculate the Parity Gap by subtracting the value of step 2 from that of step 1. ABS $(z^{T}_{CLEC\text{-}1} - {}^{c}B_{CLEC\text{-}1})$
5. Calculate the Volume Proportion using a linear distribution with slope of ¼. This can be accomplished by taking the absolute value of the Parity Gap from step 4 divided by 4; ABS $((z^{T}_{CLEC\text{-}1} - {}^{c}B_{CLEC\text{-}1}) / 4)$. All parity gaps equal or greater to 4 will result in a volume proportion of 100%.
6. Calculate the Affected Volume by multiplying the Volume Proportion from step 5 by the Total Impacted CLEC-$_1$ Volume (I_c) in the negatively affected cell; where the cell value is negative.
7. Calculate the payment to CLEC-1 by multiplying the result of step 6 by the appropriate dollar amount from the fee schedule.
8. Then, CLEC-1 payment = Affected Volume_{CLEC1} * $$from Fee Schedule

Example: CLEC-1 Missed Installation Appointments (MIA) for Resale POTS

Note – the statistical results are only illustrative. They are not a result of a statistical test of this data.

	n_I	N_C	I_c	MIA_I	MIA_C	z^T_{CLEC-1}	C_B	Parity Gap	Volume Proportion	Affected Volume
State	50000	600	96	9%	16%	-1.92	-0.21	1.71	0.4275	
Cell						z_{CLEC-1}				
1		150	17	0.091	0.113	-1.994				8
2		75	8	0.176	0.107	0.734				
3		10	4	0.128	0.400	-2.619				2
4		50	17	0.158	0.340	-2.878				8
5		15	2	0.245	0.133	1.345				
6		200	26	0.156	0.130	0.021				
7		30	7	0.166	0.233	-0.600				3
8		20	3	0.106	0.150	-0.065				2
9		40	9	0.193	0.225	-0.918				4
10		10	3	0.160	0.300	-0.660				2
										29

where n_I = ILEC observations and n_C = CLEC-1 observations

Payout for CLEC-1 is (29 units) * ($100/unit) = $2,900

Example: CLEC-1 Order Completion Interval (OCI) for Resale POTS

	n_I	n_C	I_c	OCI_I	OCI_C	z^T_{CLEC-1}	C_B	Parity Gap	Volume Proportion	Affected Volume
State	50000	600	600	5days	7days	-1.92	-0.21	1.71	0.4275	
Cell						z_{CLEC-1}				
1		150	150	5	7	-1.994				64
2		75	75	5	4	0.734				
3		10	10	2	3.8	-2.619				4
4		50	50	5	7	-2.878				21
5		15	15	4	2.6	1.345				
6		200	200	3.8	2.7	0.021				
7		30	30	6	7.2	-0.600				13
8		20	20	5.5	6	-0.065				9
9		40	40	8	10	-0.918				17
10		10	10	6	7.3	-0.660				4
										133

where n_I = ILEC observations and n_C = CLEC-1 observations

Payout for CLEC-1 is (133 units) * ($100/unit) = $13,300

2. Tier-2 Calculation For Retail Analogues

1. Tier-2 is triggered by three consecutive monthly failures of any Tier 2 Remedy Plan sub-metric.
2. Therefore, calculate monthly statistical results and affected volumes as outlined in steps 2 through 6 for the CLEC Aggregate performance. Determine average monthly affected volume for the rolling 3-month period.
3. Calculate the payment to State Designated Agency by multiplying average monthly volume by the appropriate dollar amount from the Tier-2 fee schedule.
4. Therefore, State Designated Agency payment = Average monthly volume * $$from Fee Schedule

Example: CLEC-A Missed Installation Appointments (MIA) for Resale POTS

State	n_I	n_C	l_c	MIA_I	MIA_C	$z^T_{CLEC\text{-}A}$	C_B	Parity Gap	Volume Proportion	Affected Volume
Month 1	180000	2100	336	9%	16%	-1.92	-0.21	1.71	0.4275	
Cell						$z_{CLEC\text{-}A}$				
1		500	56	0.091	0.112	-1.994				24
2		300	30	0.176	0.100	0.734				
3		80	27	0.128	0.338	-2.619				12
4		205	60	0.158	0.293	-2.878				26
5		45	4	0.245	0.089	1.345				
6		605	79	0.156	0.131	0.021				
7		80	19	0.166	0.238	-0.600				9
8		40	6	0.106	0.150	-0.065				3
9		165	36	0.193	0.218	-0.918				16
10		80	19	0.160	0.238	-0.660				9
										99

where n_I = ILEC observations and n_C = CLEC-A observations

Assume Months 2 and 3 have the same affected volumes. Payout 99 units * $300/unit = $29,700.

If the above example represented performance for each of months 1 through 3, then

Example: CLEC-A Missed Installation Appointments for 1Q00

State	Miss	Remedy Dollars
Month 1	X	$29,700
Month 2	X	$29,700
Month 3	X	$29,700
1Q00		$89,100